List of VM Consolidated documents of Dr. S. K. Kapoor

List 1 Different aspects of Vedic Mathematics

Aspect 38

Vedic Arithmetic syllabus

Text

01
Text of Ganita Sutras

वैदिक गणित सूत्राणिः
(1) एकाधिकेन पूर्वेण।
(2) निखिलं नवतश्चरमं दशतः।
(3) ऊर्ध्वतिर्यग्भ्याम् ।
(4) परावर्त्य योजयेत्।
(5) शून्यं साम्यसमुच्चये ।
(6) (आनुरूप्ये) शून्यमन्यत्।
(7) संकलनव्यवकलनाभ्याम्।
(8) पूरणापूरणाभ्याम्।
(9) चलनकलनाभ्याम्।
(10) यावदूनम्।
(11) व्यष्टिसमष्टिः।
(12) शेषाण्यङ्केन चरमेण।
(13) सोपान्त्यद्वयमन्तम् ।
(14) एकन्यूनेन पूर्वेण।
(15) गुणितसमुच्चयः।
(16) गुणकसमुच्चयः।

02
Text of Ganita Upsutras

वैदिक गणित उपसूत्राणिः
(1) आनुरूप्येण।
(2) शिष्यते शेषसंज्ञः।
(3) आद्यमाद्येनान्त्यमन्त्येन।
(4) केवलैः सप्तकं गुण्यात्।
(5) वेष्टनम्।
(6) यावदूनं तावदूनम्।
(7) यावदूनं तावदूनीकृत्य वर्गं च योजयेत्।

(8) अन्त्ययोर्दशकेऽपि।
(9) अन्त्ययोरेव।
(10) समुच्चयगुणितः।
(11) लोपनस्थपनाभ्याम्।
(12) विलोकनम्।
(13) गुणितसमुच्चयः समुच्चयगुणितः।

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Chapter 01

ACTUAL APPLICATIONS OF THE VEDIC SDTRAS TO CONCRETE MATHEMATICAL PROBLEMS

CHAPTER I A SPECTACULAR ILLUSTRATION For the reasons just explained immediately hereinbefore let us take the question of the CONVERSION of Vulgar fractions into their equivalent decimal form.

First Example : Case 1.

Fraction 1/19 I (say 1/19] whose denominator ends in 9. By the Current Method. By the Vedic one-line ment

Second Example : case I Let us now take another case of a similar type (say, 1/29 1/29) where too the denominator ends in 9. By the Current method :- By the Vedic one-line

Third example Case 3 1\49 1 By the current system.

ARITHMETICAL COMPUTATIONS
CHAPTER 2
MULTIPLICATION (by 'Nikhilaml etc. Sutra)

But, according to the Vedic system, the multiplication tables are not really above 6x 5. And a school-going pupil who knows simple addition and subtraction (of single-digit numbers) and the multiplication-table up to five times.

The Sutra reads : निखिलं नवतश्चरमं दशतः) which, literally translated, means ; "all from 9 and the last from 10" ! We shall give a detailed explanation, presently, of the meaning and applications of this cryptically-sounding formula. But just now, we state and explain the actual procedure, step by step

But before we actually take up the formula and explain its modus operand for multiplication, we shall just now explain a few corollaries which arise out of the 'Nikhilah' Siitra which is the subject-matter of this chapter. The First Corollary :

The first Corollary: The first corollary naturally arising out of the -यावदून तावदूनीकृत्य वर्गं च योजयेत् 'Nikhi1a7n1 Sutra reads as follows :- 5- * M. II which means :-"whatever the extent of its deficiency, lessen it still further to that very extent ; and also set up the square (of that deficiency)".

The Second Corollary. The second corollary is applicable only to a special case under the first corollary (i. e. the squaring of numbers ending in 6 and other cognate numbers). Its wording is exactly the same as that of the Sfitra which we used at the outset for the conversion of vulgar fractions into their recurring decimal equivalents (i. e. एकाधिकेन पूर्वेण). The Siitra now takes a totally different meaning altogether and, in fact, relates to a wholly different set-up and context altogether.

The F'irst case . The annexed table of products produced by the singledigit multiplier 9 gives us the necessary clue to an understanding of the Sttra :-

The Second Case : The second case falling under this category is one wherein the multiplicand consists of a smaller number of digits than the mulbiplier. This, however, is easy enough to handle ; and all that is necessarv is to fill the blank (on the left) in with the required number of zeroes and proceed exactly as before and then leave the zeroes out. Thus

The Third Case : (To be omitted during a first reading) . The third case coming under this heading is one where 1 the multiplier contains a smaller number of digits than the I multiplicand. Careful observation and study of the relevant table of products gives us the necessary clue and helps US to I' understand the correct application of the Siitra to this kind of examples.

Chapter III

MULTIPLICATION (by Urdhva-Tirak Sutra)

Having dealt in fairly sufficient detail with the application of the Nikhilam Sutra etc., to special cases of multiplication, we now proceed to deal with the ऊर्ध्वतिर्यग्भ्याम् (*Ūrdhva Tiryagbhyām*) Sutra which is the General Formula applicable to all cases of multiplication (and will also be found very useful, later on, in the division of a large number by another large number). The formula itself is very short and terse, consisting of only one compound word and means "vcrtically and crosswise". The applications of this brief and terse SCltra are nlarlifola (as will be seen again and again, later on). First we take it up in its most elementary application (namely, to Multiplication in general).

PRACTICAL APPLICATION "COMPOUND MULTIPLICATION A. Square Measure, Cubic Measure Ete. This is not a separate subject, all by itself. But it is often of practical interest and importance, even to lay people and deserves oar attention on that score. We therefore deal with it briefly. Areas of Rectangles.

PRACTICE AND PROPORTION IN COMPOUND MULTIPLICATION. The same procedure under the (Urdhva-Tiyak Satra) is readily applicable to most questions which come under the headings "Simple Practice" and "Compound Practice", wherein "ALIQUOT" parts are taken and many step of working are resorted to under the current system but wherein according to the Vedic method, all of it is mental Arithmetic,

Chapter IV

DIVISION (the Nikhilam Method)

. Having dealt with Multiplication at fairly considerable length, we now go on to Division ; and there we start with the Nikhikcm method (which is a special one). Suppose we have to divide a number of dividends (pf two digits each)

CHAPTER V

DIVISION (by the Paravartya method)

We have thus found that, although admirably suited for application in the special or particular cases wherein the divimrdigits are big ones, yct the Nikhilam method does not help US in the other cases (namely, those wherein the divisor consists of small digits). The last example (with 23 as divisor) at the end of the last chapter has made this perfectly clear. Hence the need for a formula which will cover the other cases. And this is found provided for in the Paraivartya Sara, which is a specialcase formula, which reads "Paraivartya Yojuyet" and which means "Transpose and apply".

Chapter 6

MENTAL DIVISION

(By simple argument per the Urdhva Tiryak Szitra) In addition to the Nikhim method and the Pariivartya method (which are of use only in certain special oases) there is a third method of division which is one of simple argumentation (based on the 'Urdhva Tiryak' Szitra and practically amounts to a converse thereof).

LINKING NOTE RECAPITULATION & CONCLUSION OF (Elementary) DIVISION SECTION In these three chapters (IV, V and VI) relating to Division, we have dealt with a large number and variety of instructive I examples and we now feel justified in postulating the following conclusions :- i (1) The three methods expounded and explained are, no doubt, free from the big handicap which thc current system labours under, namely, (i) the multiplication, of large numbers (the Divisors) by "trial digits9'.of the quotient at every step (with the chance of the product being found too big for the ! Dividend and so on), (ii) the subtraction of large numbers from large numbers, (iii) the length, cumbrousness, clumsiness etc, of the whole procedure, (iv) the consequent liability of the student to get disgusted with and sick of it all, (v) the resultant greater risk of errors being committed and so on ;\ (2) And yet, although comparatively superior to the process now in vogue everywhere, yet, they too suffer, in some mses, from these disadvantages. At any rate, they do not, in such cases, conform to the Vedic system's Ideal of "Short and Sweet" ; (3) And, besides, all the three of them are suitable only for some special and particular type (or types) of cases ; and none of them is suitable for general application to all cases :- (i) The 'Nikhilam' method is generally unsnitable for Algebraic divisions ; and almost invariably, the 'Parcivartya' process suits them better ; (ii) and, even as regards Arithmetical computations, the 'Nikhila' method is serviceable only when the Divisor-digits are large numbers (i.e., 6, 7, 8 or 9) and not at a11 helpful when the divisor digits are small ones (i.e. 1, 2, 3. 4 and 5) ; and it is only the

'Parcivartya' method that can be applied in the latter kind of cases ! (iii) Even when a convenient multiple (or sub-multiple) is made use of, even then there is room for a choice having to be mnde-by the pupil-as to whether the 'NikhLh' method or the 'Parcivartya' one should be preferred ; (iv) and there is no exception-less criterion by which the student tan be enabled to make the requisite final choice between the two alternative methods ; (v) and, as, for the third method (i.e. by the reversed 'Ordhva-Tiryalak' Siitra), the Algebraic utility thereof is plain enough ; but it is difficult in respect of Arithmetical calculations to say when, where and why it should be resorted to (as against the other two methods).

All these considerations (arising from our detailed comparative study of a large number of examples) add up, in effect, to the simple conclusion that none of these methods car1 be of general utility in all cases, that the selection of the most suitable method in each particular case may (owing to want of uniformity) be confusing to the student and that this element of uncertainty is bound to cause confusion. And the question therefore naturally-my, unavoidably arises as to whether the Vedic SGtras can give 11s a General Formula applicable to all cases.

And the answer is :-Yes, most certainly YES ! There is a splendid and beautiful and very easy method which conforms with the Vedic ideal of ideal simplicity all-round and whirh in fact gives us what we have been describing as "Vedic one linemental answers" ! This astounding method wc shall, however, expound in a later chapter under the captior, "Straight-Division"-which is one of the Crowning Beauties of the Vedic mathematics Sctras. (Chapter XXVII. q.v.).

CHAPTER VII

I. FACTORISATION (of Simple Quadratics)

Factorization wmes in naturally at this point, as a form of whst we have called "Reversed multiplication" and as a particular application of division. There is a lot of strikingly good material in the Vedic Siitras on this subject too, which is new to the modern mathematical world but which comes in at a very early stage in our Vedic 'Mathematics. We do not, however, propose to go into a detailed and exhaustive exposition of the subject but shall content ourselves with a few simple sample examples which will serve to throw light thereon and especially on the Sfitraic technique by which a Siitra consisting of only one or two simple words, makes comprehensive provision for explaining and elucidating a procedure yhereby a 80-called "difficult" mathematical problem (which, in the other system puzzles the students' brains) ceases to do so any longer, nay, is actually laughed at by them as being worth rejoicing over and not worrying over !

The Vedic system, however, prevcnts this kind of harm, with the aid of two small sub-Siitras which say (i) vh (Anurfipyena) and (i) amrq&,~+,~& (Adyamcidyencintyanacsntyelaa) and which mean 'proportionately' and 'the first by the first and the last by the last

CHAPTER VIII

FACTORISBTlON (of "Harder" Quadratics)

There is a class of Quadratic expressions known as Homogeneous Expressions of the second degree, wherein several letters (x, y, z etc.) figure and which are generally fought shy of by students (and teachers too) as being too "difficult" but which can be very easily tackled by means of the Adyamcidyena Sutra (just explained) and another sub-Siitra which consists of only one (compound) word, which reads =rsr and means :-"by (alternate) Elimination and Retention" Suppose we have to factorise the Homogeneous quadrat

Note :-This "Lopamstha/ipana" method (of alternate elimination and retention) will be found highly ussfill, later on in H.C.F., in Solid Geometry and in Coordinate Geometry of the straight line, the Hyperbola, the Conjugate Hyperbola, the dsymptotes etc.

Chapter ix

Factorization OF CUBICS ETC. (By Simple Argumentation e. t. c.)

We have already seen how, when a polynomial is divided by a Binomial, a Trinomial etc., the remainder can be found by means of the Remainder Theorem and how both the Quotient and the Remainder can be easily found by one or other ethod of division explained already. From this it follows that, if, in this process, the r

chapter x

HIGHEST COMMON FACTOR

In the current system of mathematics, we have two methods which are used for finding the H.C.F. of two (or more) given expressions. The first is by means of factorisation (which is not always easy) ; and the second is by a process of continuous division (like the method used in the G.C.M. chapter of Arithmetic). The latter is a mechanical process and can therefore be applied in all cases. But it is rather too mechanical and, consequently, long and cumbrous. The Vedic method provides a third method which is applicable to all cases and is, at the same time, free from this disadvantage

Chapter xi
SIMPLE EQUATIONS (FIRST PRINCIPLES)

As regards the solution of equations of various types, the Vedic sub-SCitras give us some First Principles which are theoretically not unknown to the western world but are not (in actual practice) utilised as basic and fundamental first principles of a practically Axiomatic character (in mathematical computations).

The Vedic method gives us these sub -formulae in a condensed form (like Parivartya etc.,) and enables us to perform the necessary operation by mere application thereof. The underlying principle behind all of them 1s & (Parduartya Yojayet) which means : "Transpose and adjust" The applications, however, are numerous and splendidly useful. A few examples of this kind are cited hereunder, as illustrations thereof :- (1) $2x+7=x+9$:. $2x-x=9$

Second Gelzeral Type (2) The above is the comnlonest kind (of transpositions). The second common type is one in which each side (t,he L.FI.S. and the R.H.8.) contains two Rinomia,l factors. - In general terms, let (x+a) (x$b)=(x+c) (x+d). The usual method is to work out the two n~ultiplicatior;s and do the transpositions and say :

Third Geuberul Type The third type is one whirh may be put into the general axLb p ; and, after doing all the cross-multiplication fornl : - --=- cx+d q and transposition etc.. lie get : x,bq-dp The student should (by practice) be able to assimi- aq-cp late and assume this also and do it all mentally as a single operation.

Fourth General Type m n The fourth type is of the form :- +- - 0 xfa x+b After all the L.C.M's, the cross-multiplications and the transpositions etc., are over, we get. -mh-na X= . This is simple enough and easy enough for m+n the student to assimildte ; and it should be assimilated and readily applied mentally to any case before us.

LINKING NOTE Special Types of Bpuationx The above types niay be described as General types. But there are, as in the cave of multiplications, divisions etc,, particular types which possess certain specific characteristics of a SPECIAL character which can be more easily tackled (than the ordinary ones) with the aid of certain very short SPECIAL processes what one may describe as mental one-linc methods). As already explained in a pevious context, all that the student has to do is to look for certain characteristics. spot them out, identify the particular type and apply the formula which is applicable thereto. These SPECIAL types of equations, we now go on to, in the next few chapters.

SIMPLE EQUATIONS (FIRST PRINCIPLES) As regards the solution of equations of various types, the Vedic sub-SCitras give us some First Principles which are theoretically not unknown to the western world but are not (in actual practice) utilised as basic and fundamental first principles of a practically Axiomatic character (in mathematical computations).

Chapter xiii
STMPLE EQUATIONS

We begih this section with an exposition of several special types of ~quat~ions which can be solved practically at sightwith t,he aid of a beautiful special Siitra which reads : ?@ ('8.linyam S6~riyasamucw.ye') a.nd which, in cryptic language (which renders it applicable to a large number of differelit eases) merely says : ' when the Samuccaya is the same, that Sum~tccaya is zero" i.e. it should be equxted to zero. 'Sun~uccayu' is a technical term which has several meanings (under different contexts) ; and we shdl explain them, one by one

FIRST MEANINC AND APPLICATION 'Xamuccaya' first nicans a tern1 which occurs as a common factor in all thc terms concei~l~rd.

:. x=-1 SECOND MEANING AND APPLICATION The word 'Samuccaya' has, as its second meaning, the product of the independent terms. Thus, (x+7)

THIRD MEANING AND APPLICATION 'Saniuccaya' thirdly meails the sum of the Denonlinators of twi, fiactionb Lav~ng the same (numerical) numerator. Thus 1 ' 1 + . - =0 ... 5x-2=0 2Zl 3x-1 This is axiom.zt,ie too and ilccds no elaboration.

FOURTH (: AND APPLICATION Fourthly, 'S

PIPTH MEANING AND APPLICATION (for Quadratics) With the same meaning (i.e. total) of the word %+ 'Samuccaya', t,here is a fifth kind of application possible or this Sfitra. And this has to do with Quadratic equations. None need, however, go into a panic over this. It is as simple and as easy as the fourth application : and even little children can understand and readiiy apply this Sttra in this context, as explained below. In thc two inst

Sixth meaning

With the same sense 'total' of the word 'rSa~,cuemya' but in a different application, we have the same Siitra coming straight to our rescue, in the sol~ltion of what the various text-books everywhere describe as "Harder Eq~~atioris", and deal with in a very late chapter 'thereof under that caption. In fact, the labcl "Harder" has stuck to this type of equations to such an extent that they devote a separate section thereto and the Matriculation examiners everywhere would almost seem to have made it :in invariable rule of practice to include one question of this type in their examination-papers !

Disguised formula

The above werc plain, simple cases which could bc readily ~+ccognised as belong~ng to the type under consideration. There, however, are several cases which really belong to this type but come under various kmds of disguises (thin, th~ck or ultra-thick)! But, however thick the disguise nlay be, thcrc axe simple devices by which we oan penetrate and see through the disguises and apply the 'Stinya Samuocaye' formula :-

Chapter xiii

MERGER TYPE of EASY SIMPLE EQUATIONS (by the Paravartya' method)

Having dealt with various sub-divisions under a few special types of simple equations which the Sfinyam Sdmyasarnuccaye formula helps us to solve easily, we now go on to and takc up another special type of simple equations which the Parcivartya Sutra (dealt with already in connection with Division etc) can tackle for us. This is of what may be described as the MERGER Type ; and this too includes several sub-headings nntlrr that heading.

The first type : The first variety is one in which a number of termd om thc left hand side is equated to a single term on the right hand side, in such manner that N,+N,+N, etc., (the sum of the numerators on the lcft) and (the single nunierator on ,.lie right) are the same.

DISGUISES Here too, we have often to deal with disguises, by seeing through and penetrating them, in the same way as in the previous chapter (with regard to the 'fliinya?]2 Sarnuccaye' formula).

Chapter x iv

COMPLEX MERGERS ' There is still another type-a special and complex type of equations which are usually dubbed 'harder' but which can be readily tackled with the aid of the Parcivartya Satra

Note :-The Cross-multiplication and 'Sinyam' method is SO simple, easy and straight before us here that there is no need to try any other process at all. The student may, however, for the sake of practice try the other methods also and get further verijcatiorb there from for the correctness of the answer just hereinabove arrived at.

Chapter xv

SIMULTANEOUS SIMPLE EQUATIONS

Here too, we have the GENERAL FORMULA applicable to all cases (under the 'Par6vartya' Stitra) and also the special Siitras applicable only to special types of cases. THE GENERAL FORMULA

A SPECIAL TYPE There is a special type of simultaneous simple equations which may involve big i~umbers and may therefore seem "hard" but which, owing to a certain ratio hetween the coefficients, can be readily i.e. mentally solved with the aid of the Satra &i 9 (SZnyam Anyat) (which cryptically says : If one is in ratio, the other one is Zero). An example will make the meani

A Second Special Type Thcrc is another special typc of simultaneous linear equations where the x-coefficient's and the y-coefficients arc fomld intcrchangcd. Xo elaborate multiplications etc., are needed here. The (axiomatic) Cpasfitra m-w=innlh ('~anhlufia-~~ccz~aknlanribh~,im') (which nieans "By addition and by snht,raction) gives us irn~nediately two eqi~at.ions giving thc values of (s+y) turd (x->.). And a repetition of the same

CHAPTER XVI

MISCELLANEOUS (SIMPLE) EQUATIONS.

There are other types of miscellaneous linear equations which can be treated by the Vedic Siitras. A few of them are i shown below.

FIRST TYPE I Practio~s of a particular cyclical kind are involved here. And, by the ParStvartya Sfitra, we write down the Numerator of the sum-total of all the fractions in question and equate it I to zero. nus :

SECOND TYPE A second type of such special simple equations is one where 1111 we have -+- = -+- and the factors (A, B, C ant1 L)) of AB AC AD BC ttie denominn.tora are in Arithnleticnl Progression. Tllc Sutw (Sopcintyadvayamantyam) which means "the ultimate and twice th tyadvayamantyam) which means "the ultimate and twice the penultimate" gives us the answer immediately, for instance :

THIRD TYPE A third type of equations are those where Numerator and Denominator on the L.H.S. (barring the independent terms) stand in the same ratio to each other as the entire Numerator and the entire Denominator of the R.H.S. stand to each other and these can be readily solved with the aid of the Upasfitra (subformula or corollary) maY@ (Antyayoreva) which means, "only the last terms" i.e, the absolute terms.

FOURTH TYPE Another type of special Fraction-Additions (in connection with Simple equations) is often met with, wherein the factors of the Denominators are in Arithmetical Progression or related to one another in a special manner as in SUMMATION OF SERIES. These we can readily solve with the aid of the same "Antyayoreva" Siitra [but in a different context, and in a different sense). We therefore deal with this special type here. (1) The &st sub-section of this type is one in which the factors are in AP.

FIRST SPECIAL TYPE (Reciprocals) Thie deals with Reciprocals. The equations have, under the current system, to be worked upon laboriously, before they can be solved. For example :

FOURTH SPECIAL TYPE And again, there is still another special type of Quadratics which are "harder" but which our old friends "$iiozya~ Anyat" and "Parcivartya" (Merger) can help us to solve easily. Note :-Apropos of the subject-matter of the immediately preceding sub-section (the 3rd special type), let us now consider 3 5 the equation 2 - This may look, at the outset, x+2 x+3 x+5 a like, but really is not, a quadratic equation of the type dealt with in the immediately previous sub-section (under S~nya? Anyat and S~nya~~ S6mya San~uocaye) but only a simple MERGER (because, not only is the number of terms on the R.H.S. one short of the number required but also g+ #+* It is really a case under Szinyam Anyat and Paicivartya (merger). Here, the TEST is the usual one for the merger procof

CONCLUDING LINKING NOTlE (On Quadratic Equations) In addition to the above, there are sevel*al other special types of Quadratic Equations, for which the Ve:dic Sfitras have made adequate provision and also suggested several beautifully interesting devices and so forth. But these we shall go into and deal -with, at a later -stage. Just at present, we address ourselves to o~ur next appropriate subject for this introductory and ill~st~~ative Volume namely, the solution of cubic and Biquadratic Equations etc.

CHAPTER XVII

QUADRATIC EQUATIONS

In the Vedic mathematics Sttras, CALCULUS comes in at a very early stage. As it so happens that DIFFERENTIAL calculus is made use of in the Vedic Sfitrqs for breaking a quadratic equation down at sight into two simple equations of the first degree and as we now go on to our study of the Vedic Siitras bearing on Quadratic equations, we shall begin this chapter with a breif exposition of the calculus.

Chapter 18

CUBIC EQUATIONS

1 We solve cubic equations in various ways: (i) with the aid of the Parcivartya SBtra, the LopnnaSthapana Sfitra, the formula (Ptirana-Aptirncibhnydm) which means "by the completion or nun-completion" of the square, the cube, the fourth power etc.) I (ii) by the method of Argumentation and Factorisatiori (as explained in a previoua chapter).

The Ptiraw M&hd The Piirapa method is well-known to the current system. In fact, the usually-in-vogue general formula -bf .\/b8-4ac - x= for the standard quadratic (axqbx+c =o)

Completiltg the Cubic With regard to cuhic equations, we combine the Parcivartya SQCra (as explained in the 'Division by Pmciwtya' chapter) and the Paraw sub-formula.

The Pziranu method explained in this chapter for the soluiion of cubic equations will be found of great help in factorisation ;and vice-versa. "Harder" cubic equations will be taken up later.

CHAPTER XIX

BIQUADRATIC EQUATIONS The procedures (PGruw etc.,)

Expounded in tho previous chapter for the solution of cubic equations can be equally well applied in the case of Biquadratics etc., too.

Chapter xx

MULTIPLE SIMULTANEOUS EQUATIONS.

We now go on to the solution of Simultaneous Equations involving three (or more) unknowns. The Lopam-Sthipanu Siit~a, the Antcrapya Siitra and the Par6vartya Siitra are the ones that we make use of for this purpose. FIRST TYPE

A SPECIAL TYPE There are several special types of Biquadratic equations dealt with in the Vedic Slit,ras. But we shall here deal with only one such special type and hold the others over to a later stage. - This type is one whcrcin the L.H.S. consists of the sum of the fourth powers of two Bmomiccls (and the R.H.S. gives us the equivalent thereof in the shtkfle of an arithmetical number.) The formula applicable to such cases is the +-w& (Vyasti Sanaasti) Sfitra (or the Lopana Sthdpana one) which teaches us how to use the average or the exact middle binomial for breaking the Biquadratic down into a simple quadratic (by the easy device of mutual cancellation of the old powers i.e. the xS snd the x).

MULTIPLE SIMULTANEOUS EQU4TIONS. We now go on to the solution of Simultaneous Equations involving three (or more) unknowns. The Lopam-Sthipanu Siit~a, the Antcrapya Siitra and the Par6vartya Siitra are the ones that we make use of for this purpose. FIRST TYPE

SECOND TYPE This is one wherein the R.H.S. contains significant figures in all the three equations. This can be solved by Parivrtya (CROSS-multiplication) so as to produce two derived equations whose R. H. S. consists of zero only, or by the first or the second of the methods utilised in the previous sub-section.

Chapter xxi

Simultaneous quaDRATIC EQUATIPNS .

The Sutras needed for the solutioll of simultaneous Quadratic equations have practically all been explained already. Only the actual applicational procedure, devices and modusoperanIi thereof have to be explained.

This is readily obtainable by Vilohnam (mere observation) and also because symmetrical values can always be reversed

CHAPTER XXII

FACTORISATION AND DIFFERENTIAL CALCULUS

In this Chapter the relevant **Sūtras** (*Guṇaka-Samuccaya* etc.,) dealing with successive differentiations, covering Leibnity's theorem, Maclaurin's theorem, Taylor's theorem etc., and given a lot of other material which is yet to be studied and decided on by the great mathematicians of the present-day western world, is also given.

Without going into the more abstruse details connected herewith, we shall, for the time-being, content ourselves with a very brief sketch of the general and basic principles involved and a few pertinent sample-specimens by way of illustration.

The basic principle is, of course, elucidated by the very nomenclature (i.e. the *Guṇaka-Samuccaya*) which postulates that, if and when a quadratic expression is the product of the Binomials $(x+a)$ and $(x+b)$, its first differential is the sum of the said two factors and so on (as already explained in the chapter on quadratic equations).

It need hardly be pointed out that the well-known rule of differentiation of a product (i.e. that if $y=uv$, when u and v be the function of x, $\frac{dy}{dx}=v\frac{du}{dx}+u\frac{dv}{dx}$) and the *Guṇaka-Samuccaya* Sūtras denote, connote and imply the same mathematical truth.

Let us start with very simple instances:

(1) $x^2+3x+2=\underset{a}{(x+1)}\ \underset{b}{(x+2)}$

$\therefore D_1$ (the first differential) $=2x+3=(x+2)+(x+1)=\Sigma a$

(2) $x^3+6x^2+11x+6=\underset{a}{(x+1)}\ \underset{b}{(x+2)}\ \underset{c}{(x+3)}$

$\therefore D_1=3x^2+12x+11=(x^2+3x+2)+(x^2+5x+6)$
$+(x^2+4x+3)=ab+bc+ac=\Sigma ab.$

$\therefore D_2=6x+12=2(3x+6)=2(x+1)+(x+2)+(x+3)=$
$=2(a+b+c)=2\ \Sigma a=\underline{2}\ \Sigma a$

(13) $\left.\begin{array}{l} 2x+\ \ y=3 \\ x^2+2xy=3 \end{array}\right\} \begin{array}{l} \therefore 1\frac{1}{2}x+2\frac{1}{4}y-\frac{3}{4}y^2=3 \\ \therefore\ 6x+9y-3y^2=12 \end{array}\Big\} \begin{array}{l} \therefore y^2-2y+1=0 \\ \therefore y=1 \text{ and } x=1 \end{array}$

or (ii) $\left.\begin{array}{l} 4x^2+2xy=6x \\ x^2+2ry=3 \end{array}\right\} \therefore\ 3x^2-6x+3=0 \ \therefore \left.\begin{array}{l} x=1 \\ \text{and } y=1 \end{array}\right\}$

(14) $\left.\begin{array}{l} x+y=2 \\ x^2+y^2+2x+3y=7 \end{array}\right\} \begin{array}{l} \therefore 4x+y+2y^2=7 \ \therefore 8-3y+2y^2=7 \\ \therefore 2y^2-3y+1=0 \ \therefore \left.\begin{array}{l} y=1 \\ x=1 \end{array}\right\} \text{ or } \left.\begin{array}{l} \frac{1}{2} \\ 1\frac{1}{2} \end{array}\right\} \end{array}$

(15) $\left.\begin{array}{l} 2x^2+xy+\ y^2=8 \\ 3x^2-xy+4y^2=17 \end{array}\right\} \ \therefore x^2+y^2=5$

And (by CROSS—multiplication)

$34x^2+17xy+17y^2=24x^2-8xy+32y^2$

$\therefore 10x^2+25xy-15y^2=0 \quad \therefore 2x^2+5xy-3y^2=0$

$\therefore (x+3y)(2x-y)=0 \quad \therefore x=-3y$ or $\frac{1}{2}y$

Substituting in $x^2+y^2=5$, we have

$9y^2+y^2=5$ or $\frac{5}{4}y^2=5 \quad \therefore y^2=\frac{1}{2}$ or 4

$$\therefore y=\pm\frac{1}{\sqrt{2}} \quad or \pm 2$$

and $\therefore x=\pm 3\sqrt{\frac{1}{2}}$ or $\pm\frac{1}{2}\sqrt{\frac{1}{2}}$ or ± 6 or ± 1.

N.B. :—Test for the correct sign (plus or minus).

(16) $\left.\begin{array}{l} 2x^2+xy+y^2=77 \\ 2x^2+3xy\quad\ =92 \end{array}\right\} \begin{array}{l} \therefore 184x^2+92xy+92y^2=154x^2+231xy \\ \therefore\ 30x^2-139xy+92y^2=0 \end{array}$

$\therefore (5x-4y)(6x-23y)=0 \quad \therefore x=\frac{4}{5}y$ or $\frac{23}{6}y$

$\therefore$ (By substitution).

$\left.\begin{array}{l} y=\pm 5 \\ x=\pm 4 \end{array}\right\}$ or $\begin{array}{l} 2\pm\sqrt{6/7} \\ \frac{23}{3}\pm\sqrt{7} \end{array}$

(17) $\left.\begin{array}{l} 3x^2-4xy+2y^2=1 \\ y^2-x^2\quad\ =-15 \end{array}\right\} \begin{array}{l} \therefore \text{(By subtraction)}, 4x^2-4xy+2y^2=16 \\ \therefore 2x-y=\pm 4 \end{array}$

$\therefore$ (By substitution), $4x^2\mp 16x+16-x^2=-15$

$\therefore 3x^2\mp 16x+31=0$ & so on.

(18) $\left.\begin{array}{l} 2x^2-7xy+3y^2=0 \\ x^2+xy+y^2=13 \end{array}\right\} \begin{array}{l} \therefore x=3y \text{ or } \frac{1}{2}y \\ \therefore \\ \text{and } x=\pm 3 \Big\} \text{ or } \left.\begin{array}{l} \pm\sqrt{\frac{13}{7}} \\ \pm 2\sqrt{\frac{13}{7}} \end{array}\right\} \end{array}$

(19) $\left.\begin{array}{l} 3x^2-4xy+2y^2=1 \\ y^2-x^2\quad\ =0 \end{array}\right\} \begin{array}{l} \therefore\ x=\pm y. \\ \therefore 3x^2-4x^2+2x^2=1 \ \therefore \left.\begin{array}{l} x=\pm 1 \\ y=\pm 1 \end{array}\right\} \end{array}$

or $3x^2+4y^2+2y^2=1 \quad \therefore \left.\begin{array}{l} y=\pm\sqrt{1/3} \\ \text{and } x=\pm\sqrt{1/3} \end{array}\right\}$

(20) $\left.\begin{array}{l} x^2-xy=12y^2 \\ x^2+y^2=68 \end{array}\right\}$ $\therefore\ x=4y$ or $-3y$

$\therefore$ By substitution, $17y^2=68$ or $10y^2=68$

$\therefore\ y=\pm\sqrt{2}$ or $\pm\sqrt{34/5}$

and $x=\pm\sqrt{8}$ or $\pm 3\ \sqrt{34/5}$

(21) $\left.\begin{array}{l} x^2-2xy+y^2=2x-2y+3 \\ x^2+xy+2y^2=2x-y+3 \end{array}\right\}$

(i) By *Śūnyam Anyat* $\therefore\ y=0$

Let $x-y=a$ $\therefore\ a^2-2a-3=0$ $\therefore\ a=3$ or -1

$\therefore\ x-y=3$ or ± 1.

Now, substitute and solve.

or (ii) By subtraction, $3xy+y^2=y$

$\therefore\ y=0$ or $3x+y=1$

Substitute and solve

N.B. :—The *Śūnyam Anyat* method is the best.

(22) $\left.\begin{array}{l} 3x^2+2xy-y^2=0 \\ x^2+y^2=2x(y+2x) \end{array}\right\}$ $\therefore\ x=-y$ or $\frac{1}{3}y$

$\therefore$ Substitute and solve

or (ii) By transposition,

$-3x^2-2xy+y^2=0$

This means that the two equations are not independent ; and therefore. any value may be given to y and a corresponding set of values will emerge for x !

("Harder" simultaneous Quadratics will be taken up at a later stage).

CHAPTEK XXIII

PARTIAL FRACTIONS

Another subject of very great importance in various mathematical operation8 in general and in Integral Calculus in particular is "Partial Fractions" for which the current systems have a very cumbrous procedure but which the 'Parivartya' Sltra tackles very quickly with its well-known MENTAL ONELINE answer process. We shall first explain the current method ; and, alonp-side of it, we shall demonstrate the "Parciwrtya" Sfitra application thereto. Suppose we have to express 3xa+12x+11 in the shape of Partial Fractions.

Chapter xxiv

INTEGRATION BY PARTIAL FRACTIONS

In this chapter we shall deal, briefly, with the question of INTEGRATION by means of Partial Fractions. But, before we takc it up, it will not be out of place for us to give a skeletonsort of summary of the first principles and process of integration (as dealt with by tlie ElcCdhika Satra). The original process of differentiation is, as is wellknown, a process in which we say :

CHAPTER XXV

THE VEDIC NUMERICAL CODE

It is a matter of historical interest to note that, in their mathematical writings, the ancient Sanskrit writers do not use figures (when big numbers are concerned) I in their numerical notations but prefer to use the letters of the Sanskrit (Devanagari) alphabet to represent the various numbers ! And this they do, not in order to conceal knowledge but in order to facilitate the recording of their arguments, and the derivation conolusions etc. The more so, because, in order, to help the pupil to memorise the material studied and assimilated they made it a general rule of practice to write even the most technical and abstruse text-books in Sfitras or in Verse (which is so much easier-even for the children-to memorise) than in prose (which is so much harder to get by heart and remember). And this is why we find not only theological, medical, astronomical and other such treatises but even huge big dictionaries in Sanskrit Verse ! So, from this stand-point, they used verse, Siitras and codes for lightening the buden and facilitating the work (by versifying scientific and even mathematical material in a readily assimilable form) ! The very fact that the alphabetical code (as used by them for t'his purpose) is in the natural order and can be immediately interpreted, is clear proof that the code language was resorted not for concealment but for greater ease in verificatio~~ etc., and the key has also been giver1 in its simplest form : glmft ;m, nfq qs, qrk -, smari; and a: vq"

Chapter XXVI

RECURRING DECIMALS

It has become a sort of fashionable sign of c~lltural advancement, not to say up-to-datism, for people now-a-days to talk not only grandly but also grandiosely and grandiloquently about Decimal coinage, Decimal weights, Decimal measurement6 etc. ; but there can be no denying or disguising of the fact that the western world as such--not excluding its mathematicians, physicists and other expert scientists-seems to have a tendency to theorise on the one hand on the superiority of the decimal notatlon and to fight shy, on the other, in actual practicGof decimals and positively prefer the "vulgar fractions" to them ! In fact, this deplorable state of affairs h

We may begin this part of this work with a brief reference to the well-know1 distinction betseen non-recurring decimals, recurring ones and partly-recurring ones. (i) A denominator containing only 2 or 5 as factors gives us an ordinary (i.e. non-recur~ing or non-circulating) decimal fraction (each 2, 5 or 10 contributing one significant digit to the decimal). For instance,

Chapter XXVII

STRAIGHT DIVISION

1 We now go on, at last, to the long-promised Vedic process of STRAIGHT (AT-SIGHT) DIVISION which is a simple and easy application of the DRDIIVA-TZRYAK Siitra which is capable of immediate application to all cases and which we have repeatedly been describing as the "CROWNING GEM of all" for the very simple reason that over arid above the universality of its application, it is the most eupemeand superlative manifestation of the Vedic ideal of the at-sight mental-one-line method of mathematical computation.

CHAPTER XXVIII

AUXILIARY FRACTIONS

In our exposition of vulgar fractions and decimal fractions, we have so far been making use of processes which help to give us the exact results in each case. Ad. in so doing, we have hitherto (generally) followed the current bystem whereby multiplications and divisions by powers of ten are mechanically effected by the simple device of putting the decimal point backwards or forwards (as the case may be)

CHAPTER XXIX

DIVISIBILITY AND SIMPLE OSCULATOR

We now take up the interesting (and intriguing) question as to how one can determine before-hand whether a certain given number (however long it may be) is dipisible by a certain given divisor and especially as to the Vedic pmcesses which can help us herein. The current system deals wth this subject but only In an ultra-superficial way and only in relation to what may be termed the most elementary elements thereof. Into details of these (including divisibility by 2, 5, 10, 3, 6, 9, 18, 11, 22 and so cn), we need not now enter (as they are well-known even to the mathematics-pupils at a very early stage of their mathematical study.) We shall take these for granted and &art aith the intermediate parts and then go on to the advanced portlons of the subject.

The Osculators As we have to utilise the "hs" (Ve#ulzas =Osculators) tl~roughout this subject (of divisibility), we shall begin aith a simple definition thereof and the method of their application. Owing to the fact that our familiar old friend the Ekadhzha is the first of these osculators (i.e. the positive osculator), the task becomes all the simpler and easier. Over and aboxe the huge nur111)er of purposes which the Ekddhika hss already been shown to fulfil, it has the Curtl~er merit of helping us to readily determine the divisibility (or otherwise) of a certain given d~vidend by a certain given divisor. Let us, for instance, start with our similar famlllar old friend or experimental-subject (or shall we say, "Guinea-pigs" the number 7. The student need hardly be reminded that the Ek&~%ika for 7 is derived from 7~7=49 and is therefore 6. Tlle Ekddhika is a clinching test for divisibility ; and the process by which it serves this purpose is technically called Vestana or "Osculation".

CHAPTER XXX

DIVISIBILITY AND COMPLEX MULTIPLEX OSCULATORS

The cases so far dealt with are of a simple type, involving only small divisors and consequently small osculators. What then about those wherein bigger numbers being the divisors, the osculators are bound to be correspondingly larger ? The student-inquirer's requirements in this direction form the subject-matter of this chapter. It meets the reeds in question by formulating a scheme of goups of digits which can be osculated, not as individual digits but in a lump, so to say

CHAPTER XXXI

SUM AND DIFFERENCE OF SQUARES

Not only with regard to questions arising in connection with and arising out of Pythagoras' Theorem (which we shall shortly be taking up) but also in respect of matters relating to the three fundamental Trigonometrical-Ratio-relationships (as indicated by the three formulae Sin2 O+cos2 0=1, l+tan% 6=Sec2 O and l+cota O= cosec2 6) etc., etc. we have often to deal with the difference of two square numbers, the addition of two square numbers etc., etc. And it is desirable to have the assistance of rules governing this subject and benefit by them

CHAPTER XXXII

ELEMENTARY SQUARING, CUBING ETC.

In some of the earliest chapters of this treatise, we have dealt, at length, with multiplication-devices of various sorts, and squaring, cubing etc., are only a particular application thereof. This is why this subject too found an integral place of its own in those earlier chapters (on multiplication). And yet it so happens that the squaring, cubing etc., of numbers have a particular entity and individuality of their own ; and besides, they derive additional importance because of their intimate connection with the question of the square-root, the cube-root etc., (which we shall shortly be taking up). And, consequently, we shall now deal with this subject (of squaring, cubing etc.), mainly by way of Preliminary Revision and Recapitulation on the one hand and also by way of presentation of some important new material too on the other

CHAPTER XXXIII

STRAIGHT SQUARING

Reverting to the subject of the squaring of numbers, the student need hardly be reminded t,hat the methods expounded and explained in an early chapter and even in the previous chapter are applicable only to special cases and that a General formula capable of universal application is still due. And, as this is intimately connected with a procedure known as the Dwandwa Yoga (or the Duplex Combination Process) and as this is of still greater importance and utility at the next step on the larlder, namely, the easy and facile extraction of square roots, we now go on to a brief study of this procedure.

CHAPTER XXXIV

VARGAMULA (SQUARE ROOT)

Armed with the recapitulation (in the last chapter) of the "Straight Squaring method" and the practical application of the Dwandwayoga (Duplex Process) thereto, we now proceed to deal with the Vargamda (i.e. the Square Root) on the same kind of simple, easy and straight procedure as in the case of "Straight Division".

CHAPTER XXXV

CUBE ROOTS of EXACT CUBES

(Mainly by Inspection and Argumentation) (Well-known) FIRST-PRINCIPLES (1) The lowest cubes (i.e. the cubes of the fist nine natural numbers) are 1, 8, 27, 64, 125, 216, 343, 512 and 729. (2) Thus, they all have their own distinct endings; and the is no possibility of over-lapping (or doubt as in the case of squares). (3) Therefore, the last digit of the cube root of an oxact cube is obvious :

CHAPTER XXXVI

CUBE ROOTS (GENERAL)

Having explained an interesting method by which the cube roots of exact cubes can be extracted, we now proceed to deal with cubes in general (i.e. whether exact cubes or not). As all numbers cannot be perfect cubes, it stands to reason that there should be a provision made for all cases. This, of course, there is; and this we now take up.

CHAPTER XXXVII

PYTHAGORAX THEOREM ETC.

Modern Historical Research has revealed-and all the modern historians of matl~cmatics have placed on record the historical fact that tl~e so-called "Pythogoras' Theorem" was known to the ancient Indians long long before the time of Pythagoras and that, just as although the Arabs introduced tlie Indian system of r~umerals into the \Vestern world and distinctly spoke of them as the "Hindu" numerals, yet, the Europeall importers thereof undiscerningly dubbed them as tlle Arabic numerals and they are still described everywhere under that designation, sinlilarly exactly it has happened d~at, although Pythagoras introduced his tl~eoreul to the Western matlicmatical ant1 scientific world long long afterwards, yet that Theorem continues to be known as Pythagoras' Theorem!

CHAPTER XXXVIII

APOLLONIUS' Apollonius' Theorem (sic)

is practically a direct and clcmcntary Borollary or offslloot from Pythagoras' Theorem. But, unfortunately, its proof too has been beset with the usual flaw of irksome and ~leedl~~s length and laboriousness. The usual proof is well-known and need not be reiterated here. We need only point out the Vedic method and leave it to the discerning reader to do all tlic contrasting for himself. Arid, afLer all, that is the best way. Isn't it. ?

CHAPTER XXXIX

ANALYTICAL CONICS

Analytical Conics is a vcry important branch of mathemaf tical study and has a direct bearing on practical work in various branches of mathematics. It is in the fitness of things, therefore, that Analytical Conics should find an important and predominating position for itself in the Vedic system of mathematics (as it actually does). A few instances (relating to certain very necessary and very important points connected with Analytical Conics) are therefore given here und

CHAPTER XL

MISCELLANEOUS MATTERS

There are also various subjects of a miscellaneous character which are of great practical interest not only to mathematicians and statisticians as such but also to ordinary people in the ordinary course of their various businesses etc., which the modern system of accounting etc., does scant justice to and in which the Vedic Siitras can be very helpful to them. We do not propose to deal with them now, except to .name a few of them : (1) Subtractions ; (2) Mixed additions and subtractions ; (3) Compound additions and subtraction@; (4) Additions of Vulgar Fractions etc ; (6) Comparison of Fractions ; (6) Simple and compound practice (without taking Aliquot parts etc.) (7) Decimal operations in all Decimal Work ; (8) Ratios, Proportions, Percentages, Averages etc. ; (9) Interest ; Annuities, Discount etc ; (10) The Centre of Gravity of Hemispheres etc ; (11) Transformation of Equations ; and (12) Dynamics, Statics, Hydrostatics, Pneumatics etc., Applied Mechanics etc., etc. N.B. : -There are some other subjects, however, of an important character which need detailed attention but which (owing to their being more appropriate at a later stage) we do not now propose to deal with but which, at the same time, in view of their practical importance and their absorbingly interesting character, do require a brief description. We deal with them, therefore, briefly hereunder. Solids, Trigonometry, Astm~y Etc. In Solid Geometry, Plane Trigonometry, Spherical Trigonometry and Astronomy too, there are similarly huge masses of Vedic material calculated to lighten the mathematics students' burden. We shall not, however, go here and now into a detailed disquibition on such matters but shall merely name a few of the important and most interesting headings under which these subjects may be usefully sorted : (1) The Trigonometrical Functims and their interrelationships ; etc. (2) Arcs and chords of circles, angles and sines of angles etc ;. (3) The converse i.e. sines of angles, the angles themselves, chords and arcs of circles etc ; (4) Determinants and their use in the Theory of Equations, Trigonometry, Conics, Calculus etc ; (5) Solids and why there can be only five regular Poly hedrons ; etc., etc. (6) The Earth's daily Rotation on its own axis and her annual relation around the Sun ; (7) Eclipses ; (8) The Theorem (in Spherical Triangles) relating to the product of the sines of the Alternate Segments i.e. about : Sin BD Sin CE Sin AF-l and sDC 'SEA STB- (9) The value of 11 (i.e. the ratio of the circumference of a circle to its Diameter). N.B. :-The last item, however, is one which we would like to explain in slightly greater detail. - 11 Actually, the value of , is given in the well-known 10 Anustub metre and is couched in the Alphabetical Code-Language (described in an earlier chapter) : It is so wordcd that it can bear three different meaningsall of them quite appropriate. The first-is a hymn to the Lord 8ri Kysna ; the second is similarly a hymn in praise of the C Lord Shri Shankara ; and the third is a valuation of gto 32 10 places of Decimals ! (with a "Self-contained master-key" for I extending the evaluation to any number of decimal places ! As the student (and especially the non-Sanskrit knowing student) is not likely to be interested in and will find great ditliculty in underdtanding the puns and other literary beauties of the verse in respect of the first two meanings but will naturally feel interested in and can easily follow the third meaning, we give only that third one here : - -- 11-.3141692653589793 10 9384626431831792.. .I on which, on understanding it, Dr. V. P. Dalal (of the Heidelburg University, Germany) felt impelled-as a mathematician and physicist and also as a Sanskrit scholar-to put on record his comment as follows : "It shows how deeply the ancient Indian mathematicians penetrated, in the subtlety of their calculations, even when the Greeks had no numerals above 1000 and their multiplications were so very complex, vhich they performed with the help of the counting frame by adding so many times the multiplier ! 7 x5 could be done by adding 7 on the counting frame 5 times !" etc., etc. !

RECAPITULATION AND CONCLUSION

In these pages, we have covered a large number of branches of mathematics and sought,by comparison and rontrast,tom~ke the exact position clear to all seekers after knowledge. Arithmetic and Algebra being the basis on which all mathematical operations have to depend, it was and is both appropriate and inevitable that, in an introductory and preliminary volume of this particular character, Arithmetic and Algebra should have received the greatest attention in this treatise. But this is only a kind of preliminary "PROLEGOMENA" and SAMPLE type of publication and has been intended solely for the purpose of giving our readers a foretaste of the delicious delicacies in store for them in the volumes ahead.l If this volume achieves this purpose and stimulates the reader's interest and prompts him to go in for a further detailed study of Vedic Mathematics we shall feel more than amply rewarded and gratified thereby

Vedic Arithmetic topics

1. Arithmetic – numbers – Arithmetic operations
2. Basic arithmetic operations :
   a. Addition and subtraction
   b. Multiplication and division
   c. Powers and roots
3. Numbers 1 to 1000
   a. Numerals 1 to 9
   b. Perfect numbers 6, 28, 496

   Powers value uptill 1000

   c. Squares $1^2, 2^2, 3^2$, --- 31^2,
   d. Cubes $1^3, 2^3$, ….10^3,
   e. Power 4 $1^4, 2^4$, …..10^4
   d. Power 5 $1^5, 2^5, 3^5$…
   e. Power 6 $1^6, 2^6$,
   f. Power 7 $1^7, 2^7$,
   g. Power 8 $1^8, 2^8$,
   h. Power 9 $1^9, 2^9$,
   i. Power 10 1^{10}

4. Primes

- Uptill 1-10	4	4
- Uptill 5-100	21	25
- Uptill 101-200	21	46
- Uptill 201-300	16	62
- Uptill 301-400	16	78
- Uptill 401-500	17	95
- Uptill 501-600	14	109
- Uptill 601-700	16	125
- Uptill 701-800	14	139
- Uptill 801-900	15	154
- Uptill 901-1000	14	168

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659 661 673 677 683 691

701 709 719 727 733 739 743 751 757
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769 773 787 797

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863 877 881 883 887

907 911 919 929 937 941 947 953 967
971
977 983 991 997

Syllabus of Vedic Mathematics prescribed

in Vedic Maths Trainings

1. Number System,
2. Addition
3. Subtraction
4. Mixed Calculation of Addition and Subtraction
5. Tables upto 500,
6. Multiplication of different Methods
7. Sum of Products
8. Difference of Products
9. Mixed Calculation of Products,
10. Division
11. Divisibility
12. Determination of quotient using divisibility rule
13. Square using different method
14. Addition of Squared Numbers
15. Subtraction of Squared Numbers
16. Mixed calculations of squared numbers,
17. Cube of different methods
18. Addition of cubes, difference of cube numbers, square root

19. cube root

20. Products in Algebraic Expressions

21. Mixed calculation in Algebraic expressions

22. Square of algebraic expression

23. Cube of Algebraic expression

24. Square root and cube root in algebraic expression

25. Solution of simultaneous linear equations

26. factorization of quadratic cubic algebraic expressions

27. Solution of simple equations, roots of quadratic and cubic equations

28. Division in Arithmetic and Algebra

29 .Baudhayan Numbers

30.Formulations of trigonometry formulae

31. Coordinate geometry

32. Differentiation of Composite function

33. Integration of Composite function

34. Inverse of Determinant

35. Solution of equations using determinant method,

36 Similarity in Geometry

37 Mensuration
